Estimating the Primary Beam of the e-MERLIN Array in L-Band

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Introduction

An interferometer observes visibilites of the sources that lie within the beams of all the antennas in the array and represents the ultimate restriction in wide field imaging. The synthesised image brightness is not the true sky brightness but is attenuated as a function from the *pointing centre* depending on the antenna directivity and can be transformed as;

$$\widetilde{I}(\Theta) = A(\Theta)I(\Theta) \quad (1)$$

where $A(\Theta)$ is the primary beam function (of the array), $I(\Theta)$ is the true sky brightness distribution and $\widetilde{I}(\Theta)$ represents the synthesised image. If all the antennas have identical beamshapes (as in the case of the VLA) then $A(\Theta)$ is identical to that of any one antenna. However, in an interferometer with dissimilar antennas the beamshape is some complicated weighted combination of all of them. In the MERLIN array most of the antennas can be treated as being roughly the same (25m diameters) with two exceptions - the Cambridge dish (32m diamter) and the Lovell dish (76m diameter) at Jodrell Bank. To determine the power beamshape of a single baseline of dissimilar antennas requires knowledge of the voltage polar diagram $V(\theta)$ for each antenna. The power beamshape of the antenna pair in a multiplying interferometer is simply the product of the two voltage beams.

Beamshapes

The beamshape of a uniformly illuminated circular aperture can be derived in terms of Bessel functions and predicts the Airy disk function familier to optical telescope users. However in the case of radio telescope dishes, the illumination function is often not uniform but altered, by way of a modification of the prime focus collecting horn (or secondary reflector if Cassegrain) such that radiation received from the edges of the dish is deliberately attenuated. This is called *aperture tapering* and acts to reduce undesirable beam sidelobes which can receive flux from unwanted directions. Consequently the application of the Airy disk equation assocated with a simple uniformly illuminted circular aperture is only approximately correct for radio antennas. In most cases the aperture tapering is not well defined and therefore determination of the antenna beamshape is best carried out experimentally. This can be achieved by scanning the telescope beam across a point source of unchanging flux density and recording attenuations relative to the pointing centre. The procedure has been carried out for the VLA (25m) antennas and the results fitted to a polynomial function by R Perley[AIPS];

$$P(\theta) = 1 + G_1(\theta f)^2 + G_2(\theta f)^4 + G_3(\theta f)^6 \quad (2)$$

where f is the observing frequency in GHz, θ is the off-axis angle from pointing centre and the best LS fit parameters are given in Table 1.

Parameter	Value at 1.465 GHz
G_1	-1.343 E-3
G_2	6.579 E-7
G_3	-1.186 E-10

Table 1: L-Band Perley Fit Parameters

The Half Power Beam Width (HPBW) or Full Width at Half Maximum (FWHM) for the 25m telescopes given by the Perley fit at 1.465 GHz was determined to be 29.6 arcminutes by plotting the above function. When the HPBW is estimated using;

$$\theta_{\frac{1}{2}} \approx k\frac{\lambda}{D} \tag{3}$$

where D is the antenna diameter, the best fit value for k is approximately 1.05. This is close to that predicted by uniform illumination and is expected because the VLA antennas have little aperture tapering, however the prime focus assembly does cause some deviation. The beamshape of a tapered aperture can be derived by taking the 2 dimensional Fourier transform of the autocorrelation of the aperature distribution, implying that the beamshape of a Gaussian tapered antenna will also be Gaussian. Thus a tapered antenna beam can usually be well modelled by a Gaussian - at least down to the HPBW. Figure 1 illustrates a Gaussian fit to the Perley regression. The form of the Gaussian function is;

$$P(\theta) = W \exp\left(-\frac{\theta^2}{2\sigma^2}\right) \tag{4}$$

where W is a scaling factor (can be taken to be unity for a relative power beam) and σ is propotional to the Half Power Beam Width by;

$$\sigma^2 = \frac{\theta_{\frac{1}{2}}^2}{8\ln 2} \tag{5}$$

where $\theta_{\frac{1}{2}}$ is the HPBW. The relative power beam of a single antenna (within the HPBW) can therefore be represented by;

$$P(\theta) = W \exp\left(-\frac{\theta^2 4\ln 2}{\theta_{\frac{1}{2}}^2}\right) \tag{6}$$

where θ is the radius from the pointing centre in radians. Figure 1 illustrates the Gaussian approximation applied to the VLA beam and compares it to the measured function described by Perley. The voltage beam (modulus) can be approximated (out to a certain maximum angular range) by taking the square root of the above function yielding:

$$V(\theta) = \sqrt{W} \exp\left(-\frac{\theta^2 2\ln 2}{\theta_{\frac{1}{2}}^2}\right) \tag{7}$$

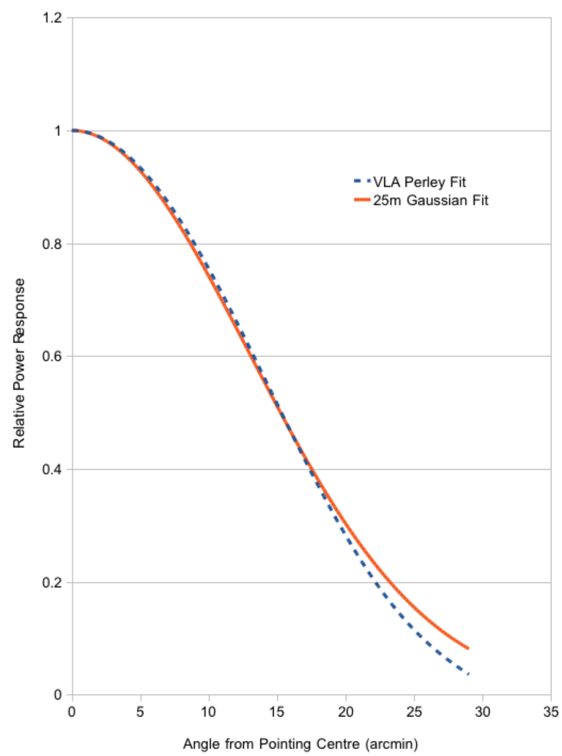

Figure 1: Perley Fit for VLA Primary Beam compared with a Gaussian Approximation

The primary power beam of any baseline pair in a multiplying interferometer is the product of the two voltage beams of the two antennas $V_A(\theta) \times V_B(\theta)$. Applying the Gaussian approximation yields the following expression for estimating the HPBW ($\theta_{\frac{1}{2}AB}$) for any baseline pair of antennas with known HPBWs:

$$\theta_{\frac{1}{2}AB} = \left[\frac{2}{\theta_{\frac{1}{2}A}^{-2} + \theta_{\frac{1}{2}B}^{-2}}\right]^{\frac{1}{2}} \tag{8}$$

In cases where the half power beam widths are not known for all the antennas (as for Lovell and Cambridge antennas of the MERLIN array) an approximation can be made using Equation 3. However, because the Lovell dish is more tapered (under illuminated) than the 25m dishes, it is likely that value of k parameter in Equation 3 will be larger - meaning a wider beam. There are effectively 3 antenna diameters in the MERLIN array (25m, 32m and 76m) which gives rise to 4 types of baseline pairings. Because there are multiple baseline pairs in the e-MERLIN array, the total primary power beam must be correctly weighted according to each baseline pair. Each power beam pair P_i is scaled according to:

$$B_i = \frac{W_i}{\sum W_i} P_i \tag{9}$$

The total primary power beam for the array is then simply:

$$P_T = \sum B_i \tag{10}$$

The power weighting for each beam pair W_i is the geometric product of their sensitivity weightings W_A and W_B (given in Table 2) because it is their voltage beams that are multiplied - not their power beams. The geometrically weighted means for all possible baseline pair combinations in the e-MERLIN array are described in Table 3. These combinations can be grouped and normalised into the 4 similar beamshape pairs and then summed yielding 4 factors which represent the relative weighting to apply to each similar type of beamshape function. Using the above weighting data, the power beamshape for the e-MERLIN array is estimated to be equal to:

$$P_M(\theta) = 0.15P_{25} + 0.58P_{76*25} + 0.11P_{25*32} + 0.16P_{76*32} \tag{11}$$

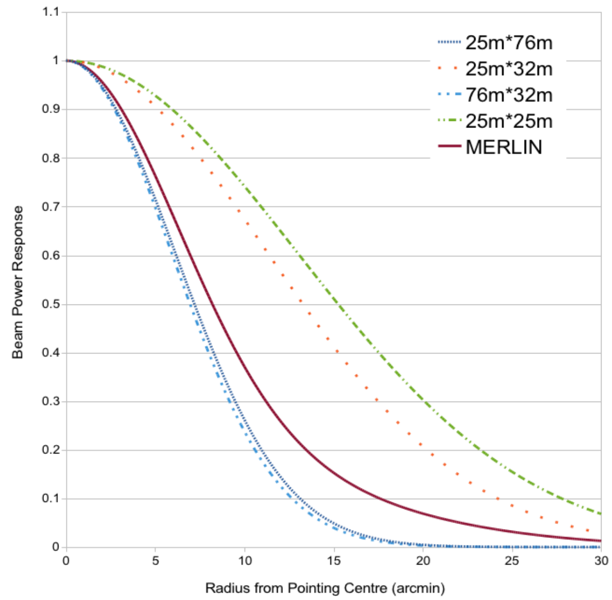

Figure 2: Predicted Primary Beams for Antenna Pairs and the MERLIN array at 1.42 GHz

It is clear that the baselines involving the Lovell antenna (i.e. 76*25 and 76*32) dominate the beamshape contributing over 74% of the power. Consequently the beamwidth of the MERLIN array will be closer to the beamwidth of the Lovell baseline pairs than to baseline pairs between other antennas (see Figure 2). The HPBW of the e-MERLIN beam is therefore heavily dependant on the Lovell Baseline weightings and hence the Lovell antenna weighting. Furthermore because of the beamshape is Lovell dominated it is important to determine the beamwidth of Lovell telescope. This can be estimated by measuring flux densities of point sources at various angles from the pointing centre in a MERLIN observation field and comparing them with corrected fluxes from the same set of observations, but omitting all Lovell baselines.

Several problems exist in attemping to measure flux attenuations:

- Most sources are resolved by MERLIN implying some flux loss

Antenna	Diameter (m)	Relative Sensitivity (W)
Defford (DE)	25	0.61
Cambridge (CA)	32	1.74
Knockin (KN)	25	0.73
Darhnall (DA)	25	1.00
Mark II (MK)	25	1.00
Lovell (LO)	76	50.0
Tabley (TA)	25	0.77

Table 2: Antennas in the MERLIN array. The relative power sensitivities are taken from the MERLIN handbook.

	Sensitivity:	*1.74*	*50*	*1.0*	*1.0*	*0.73*	*0.77*
Sensitivity	Antenna Name	*CA*	*LO*	*MK*	*DA*	*KN*	*TA*
0.61	*DE*	1.03	5.52	0.78	0.78	0.67	0.69
0.77	*TA*	1.16	6.2	0.88	0.88	0.75	-
0.73	*KN*	1.13	6.04	0.85	0.85	-	-
1.0	*DA*	1.32	7.07	1.00	-	-	-
1.0	*MK*	1.32	7.07	-	-	-	-
50	*LO*	9.33	-	-	-	-	-

Table 3: Geometrically averaged weightings for all 21 baseline-pair combinations in the e-MERLIN array

- The amount of resolved flux loss due to losing the small Lovell baselines is unknown

In an effort to establish the Lovell HPBW, 22 partially or totally unresolved by the VLA were chosen from the Morrison Catalogue as candidates within the Hubble Deep Field - North region and imaged both with and without the Lovell antenna using a set of MERLIN observations made in 1996 (at that time the Wardle antenna was still operating so the relevant weighting factor was included to account for this). The removal of these baselines reduces the sensitivity to about 25% of the normal sensitivity for MERLIN and consequently the unresolved sources had to be relatively bright as well as being distributed over a good angular range from the pointing centre. Both peak and integral flux densities were measured using JMFIT in the AIPS suite. Some sources were either too resolved at MERLIN resolutions or too weak to be usuable without the Lovell antenna, so were ignored. However one very bright unresolved source lying some 5.5 arcminutes from the pointing centre (which is about the expected radius to half power for a 76m diameter antenna) could be used to anchor the beamwidth with a few additional points scattered between 4 and 7 arcminutes. The flux densities measured without the Lovell baselines were corrected upwards by assuming that the other antennas had beamwidths equal to $1.05\lambda/D$ and by use of the appropriate antenna weighting scheme discussed above. It was then possible to adjust the assumed beamwidth of the Lovell telescope (by varying the value of k in Equation 3) until the predicted MERLIN beamwidth gave rise to the attenuations measured in observations that included the Lovell baselines. It was found that the goodness of fit was poor which may indicate that the Lovell beam deviates slightly from a Gaussian approximation beyond its HPBW as did the VLA antennas. A more rigorous treatment of the data may find a better type of fit, a polynomial being the most obvious similar to Equation 2, however because several extra free parameters would then exist, a larger dataset would be essential. The best value of k_{LO} which correctly predicted flux at-

Radius from PC	Beam corrected S mJy (No LO)	S mJy (with LO)	Attenuation	Predicted ($k_{LO} = 1.13$)
4.0'	319	272	0.85	0.84
4.6'	144	165	1.14	0.8
4.8'	584	472	0.8	0.78
5.5'	1666	1207	0.72	0.72
5.6'	227	256	1.13	0.71
5.7'	204	201	0.98	0.71
6.5'	276	156	0.56	0.64
6.8'	210	81	0.38	0.61
6.9'	327	198	0.6	0.61

Table 4: Observations of Sources made with and without Lovell Baselines at 1.42GHz. Several sources have clearly had some flux resolved away making any goodness of fit highly uncertain. Only one source was truely unresolved at 5.5' from the pointing centre (PC).

tenuations for the largest number of points including the brightest source, was found to be approximately 1.13, but to account for the probable Gaussian overestimate beyond the HPBW it could be argued that this value should be reduced somewhat to compensate. A crude method to determine the likely amount of reduction is to scale the Perley polynomial directly such that the HPBW of the Lovell given by $1.13\lambda/D$ fits. Calculating baseline pairs using both methods generates the estimates given in Table 5. A reduced value of k=1.1 in a Gaussian approximation produces similar results as the scaled Perley polynomial at 1.42 GHz and could be adopted for a lower limit estimation.

Primary beam as a function of observing frequency

The band coverage of e-MERLIN (in L-band) ranges from 1.42 GHz to 1.75 GHz sampled over several hundred narrow channels. Consequently each different frequency 'sub-band' has its own associated beamwidth - narrower at the high frequency end of the range than at the low frequency end. The width of the e-MERLIN primary beam can be modified by increasing or decreasing the relative weighting of the Lovell baselines (compared to the other baselines) because they tend to dominate the overall beamwidth. The Lovell antenna should arguably be weighted slightly less at the higher frequencies than at the lower frequencies because its efficiency drops as a function of increasing frequency more so than for the other antennas in the array. Such weighting can increase the relative contributions of other baselines thereby increasing the primary beamwidth of the array. Using the weighting scheme given in Table 3 but altering the Lovell weighting generates the predictions given in Table 6. Weighting can be applied so that the HPBW is more consistant between bands by interpolating weights for each IF. The main cost is sensitivity, however it may be useful in minimising the number of pointings when mosaicing a large patch of sky.

References

[AIPS] AIPS Explain PBCOR Explain File, NRAO...

Baseline Pair	HPBW (Scaled Perley Fit)	HPBW (Gaussian k=1.13)
25m*25m	30.4'	30.4'
76m*25m	14.0'	14.4'
32m*25m	26.2'	26.2'
76m*32m	13.6'	14.0'

Table 5: Estimated HPBWs for 25m, 32m and 76m antenna baseline pair combinations at 1.42 GHz.

Frequency	$HPBW_{76*25}$	$HPBW_{76*32}$	$HPBW_{25*25}$	$HPBW_{32*25}$	Lovell Weight	LO-BL Power	$MERLIN_{HPBW}$
1.42 GHz	14.0'	13.6'	30.4'	26.2'	30	70%	16.6'
					50	75%	16.0'
					75	78%	15.7'
					100	81%	15.5'
1.75 GHz	11.4'	11.0'	24.8'	21.6'	30	70%	13.6'
					50	75%	12.0'
					75	78%	12.8'
					100	81%	12.6'

Table 6: Best estimate Half Power Beam Widths for the e-MERLIN array at opposite ends of the band. Decreasing the Lovell antenna weighting widens the primary beam of the array and reduces the relative power received by LO baselines.